Schwartz 3-5(a)

$$\mathcal{L} = -\frac{1}{2}\phi\Box\phi + \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

Integrate by parts:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial_\mu\phi) + \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

$$\partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} = \partial_\mu^2\phi = \Box\phi, \quad \frac{\partial\mathcal{L}}{\partial\phi} = m^2\phi - \frac{\lambda}{6}\phi^3.$$

EOM: $\Box\phi - m^2\phi + \frac{\lambda}{6}\phi^3 = 0$

ansatz: $\phi = c$, then

$$\frac{\lambda}{6}\phi^{\cancel{3}\,2} = m^2\cancel{\phi}$$

$$\phi^2 = \frac{6}{\lambda}m^2,$$

$$\boxed{\phi = \pm\sqrt{\frac{6}{\lambda}}\,m.}$$

GND: $\phi = -\sqrt{\frac{6}{\lambda}}\,m.$

(b) $\phi \to -\phi$ swaps the gND state.

(c) $\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial_\mu \phi + \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$

$$= \frac{1}{2}\partial_\mu \pi \partial_\mu \pi + \frac{1}{2}m^2[c+\pi]^2 - \frac{\lambda}{4!}[c+\pi]^4$$

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \pi)} = \Box \pi$$

$$\frac{\partial \mathcal{L}}{\partial \pi} = m^2[c+\pi] - \frac{\lambda}{6}[c+\pi]^3.$$

EOM: $\Box \pi - m^2[c+\pi] + \frac{\lambda}{6}[c+\pi]^3 = 0.$

$\pi=0$ satisfies this eq with $c = \pm\sqrt{\frac{6}{\lambda}}\, m.$

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